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## Flavor symmetry for strong phases and determination of $\beta_s, \Delta\Gamma$ in $B_s \rightarrow J/\psi\phi$

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Studies of angular and time distributions and CP-violating observables in  $B_s \rightarrow J/\psi\phi$  decays yield a space of parameters which can be considerably reduced if relative strong phases among different amplitudes are specified. We show that the relations between  $B_s \rightarrow J/\psi\phi$  and  $B^0 \rightarrow J/\psi K^{*0}$  amplitudes given by flavor symmetry [actually U(3) rather than SU(3)] are likely to be quite reliable, and hence the use of strong phases from  $B^0 \rightarrow J/\psi K^{*0}$  in the analysis of  $B_s \rightarrow J/\psi\phi$  is justified. We point out the potential advantage of using helicity angles over transversity angles, and comment on a way to measure a sizable CP-violating phase independent of strong phases.

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The decay  $B_s \rightarrow J/\psi\phi$  is of great interest in the study of CP violation. In the Standard Model in which CP violation is generated purely by phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, the CP asymmetry in this process is expected to be small, as it should be governed by the  $B_s$ - $\bar{B}_s$  mixing phase  $\phi_M = -2\beta_s$ , where  $\beta_s = \text{Arg}(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*) = \lambda^2\eta \simeq 0.02$  [1], with  $\lambda = 0.2255 \pm 0.0019$  [2] and  $\eta \simeq 1/3$  parameters in the CKM matrix.

Recently both the CDF Collaboration [3] and the D0 Collaboration [4], working at the Fermilab Tevatron collider, have reported fits to angular and time distributions of flavor-tagged  $B_s \rightarrow J/\psi\phi$  decays which favor larger values of  $2\beta_s$ . (Earlier reports by CDF and D0 which did not apply flavor tagging can be found in Refs. [5,6] and [7].) In the absence of information about relative strong phases between different amplitudes, these solutions have a two-fold ambiguity which can be eliminated with strong phase information [8] from the decay  $B^0 \rightarrow J/\psi K^{*0}$  using flavor symmetry. The D0 analysis uses this information, while CDF finds consistency with it.

The present note examines the robustness of this assumption, and finds it to be valid to a sufficient degree that the two-fold ambiguity indeed can be eliminated. Our approach is the inverse of that of Ref. [9], which suggested making use of strong phase information in  $B_s \rightarrow J/\psi\phi$  (assuming Standard Model weak phases) to resolve a discrete ambiguity in the sign of  $\cos 2\beta$  in  $B^0 \rightarrow J/\psi K^{*0}$ . In passing we make some remarks about the angular and time dependence of the  $B_s \rightarrow J/\psi\phi$  observables. In order to avoid a potential bias due to an uncertainty in angular acceptance, we suggest using a

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set of angles differing from those used by CDF and D0. We offer the possibility that a large value of  $2\beta_s$  could show up in time oscillations which do not depend on strong phases.

In order to specify the two-fold ambiguity resolved by strong phases, we first review the angular and time dependence in decays  $M \rightarrow V_1 V_2$ , where  $M$  is a spinless meson,  $V_{1,2}$  are vector mesons, and  $V_1 \rightarrow \ell^+ \ell^-$ ,  $V_2 \rightarrow P_A P_B$ . Here  $\ell^+$  and  $\ell^-$  are leptons, while  $P_A$  and  $P_B$  are pseudoscalar mesons. We use the *transversity basis* [10] and its application to these decays in Ref. [11].

In the rest frame of  $V_1$  let the direction of  $V_2$  define the  $x$  axis. Let the plane of the  $P_A P_B$  system define the  $y$  axis, with  $p_y(P_A) > 0$ , so the normal to that plane (taking a right-handed coordinate system) defines the  $z$  axis. For  $\phi \rightarrow K^+ K^-$  and  $K^{*0} \rightarrow K^+ \pi^-$ , we shall take  $P_A \equiv K^+$ . A unit vector  $n$  in the direction of the  $\ell^+$  in  $V_1$  decay may be defined to have components

$$(n_x, n_y, n_z) = (\sin \theta_T \cos \phi_T, \sin \theta_T \sin \phi_T, \cos \theta_T) , \quad (1)$$

thereby defining the polar and azimuthal transversity angles  $\theta_T$  and  $\phi_T$ . A third angle  $\psi$  is defined as that of  $P_A$  in the  $V_2$  rest frame relative to the helicity axis (the negative of the direction of  $V_1$  in that frame).

The decay of a spinless meson  $M$  to  $V_1 V_2$  is characterized by three independent amplitudes [12], corresponding to linear polarization states of the vector mesons which are either longitudinal (0), or transverse to their directions of motion and parallel ( $\parallel$ ) or perpendicular ( $\perp$ ) to one another. The states 0 and  $\parallel$  are P-even, while the state  $\perp$  is P-odd. When  $V_1$  and  $V_2$  are eigenstates of C with the same eigenvalue (as in the case  $V_1 = J/\psi$ ,  $V_2 = \phi$ ), the properties under P are the same as those under CP. We denote the invariant amplitudes by  $A_0$ ,  $A_{\parallel}$ ,  $A_{\perp}$ , using the normalization

$$|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2 = 1 . \quad (2)$$

The corresponding distribution in  $\vec{\rho}_T \equiv (\theta_T, \phi_T, \psi)$  is [11]

$$\begin{aligned} \frac{d^4 \Gamma[M \rightarrow (\ell^+ \ell^-)_{V_1} (P_A P_B)_{V_2}]}{d \cos \theta_T d \phi_T d \cos \psi dt} &\propto \frac{9}{32\pi} [|A_0|^2 f_1(\vec{\rho}_T) + |A_{\parallel}|^2 f_2(\vec{\rho}_T) + |A_{\perp}|^2 f_3(\vec{\rho}_T) \\ &+ \text{Im}(A_{\parallel}^* A_{\perp}) f_4(\vec{\rho}_T) + \text{Re}(A_0^* A_{\parallel}) f_5(\vec{\rho}_T) + \text{Im}(A_0^* A_{\perp}) f_6(\vec{\rho}_T)] , \end{aligned} \quad (3)$$

where

$$\begin{aligned} f_1(\vec{\rho}_T) &\equiv 2 \cos^2 \psi (1 - \sin^2 \theta_T \cos^2 \phi_T) , \quad f_2(\vec{\rho}_T) \equiv \sin^2 \psi (1 - \sin^2 \theta_T \sin^2 \phi_T) , \\ f_3(\vec{\rho}_T) &\equiv \sin^2 \psi \sin^2 \theta_T , \quad f_4(\vec{\rho}_T) \equiv -\sin^2 \psi \sin 2\theta_T \sin \phi_T , \\ f_5(\vec{\rho}_T) &\equiv \frac{1}{\sqrt{2}} \sin 2\psi \sin^2 \theta_T \sin 2\phi_T , \quad f_6(\vec{\rho}_T) \equiv \frac{1}{\sqrt{2}} \sin 2\psi \sin 2\theta_T \cos \phi_T . \end{aligned} \quad (4)$$

We now describe the time-dependence of the decay for  $M = B_s$ ,  $V_1 = J/\psi$ ,  $V_2 = \phi$ , following discussions in Refs. [8, 13–15]. The mass-eigenstate combinations of  $B_s$  and  $\overline{B}_s$ , which have definite time-development properties, are

$$|B_{sL}\rangle = p|B_s\rangle + q|\overline{B}_s\rangle , \quad |B_{sH}\rangle = p|B_s\rangle - q|\overline{B}_s\rangle , \quad (5)$$

Table I: Comparison of BaBar (Ba) [19], Belle (Be) [20], Heavy Flavor Averaging Group (HFAG) averages of these values (H) [21], and CDF (C) [22] magnitudes and phases of  $B^0 \rightarrow J/\psi K^{*0}$  decay amplitudes. The CDF values are not included in the HFAG averages.

	$ A_0 ^2$	$ A_{\parallel} ^2$	$\delta_{\parallel}$	$\delta_{\perp}$
Ba	$0.556 \pm 0.009 \pm 0.010$	$0.211 \pm 0.010 \pm 0.006$	$-2.93 \pm 0.08 \pm 0.04$	$2.91 \pm 0.05 \pm 0.03$
Be	$0.574 \pm 0.012 \pm 0.009$	$0.231 \pm 0.012 \pm 0.008$	$-2.89 \pm 0.09 \pm 0.01$	$2.94 \pm 0.06 \pm 0.01$
H	$0.56 \pm 0.01$	$0.219 \pm 0.009$	$-2.91 \pm 0.06$	$2.92 \pm 0.04$
C	$0.569 \pm 0.009 \pm 0.009$	$0.211 \pm 0.012 \pm 0.006$	$-2.96 \pm 0.08 \pm 0.03$	$2.97 \pm 0.06 \pm 0.01$

with  $|p|^2 + |q|^2 = 1$ . In the limit in which  $|\Delta\Gamma/\Delta m| \ll 1$  (a good approximation here),  $q/p \simeq e^{2i\beta_s}$ , a pure phase. Heavy and light eigenstate masses and widths are  $(m_H, m_L)$  and  $(\Gamma_H, \Gamma_L)$ ; their differences are  $\Delta m_s \equiv m_H - m_L$ ,  $\Delta\Gamma \equiv \Gamma_L - \Gamma_H$  (note the sign). The average of CDF and D0 measurements gives  $\Delta m_s = 17.78 \pm 0.12 \text{ ps}^{-1}$  [16]. In the Standard Model limit of small CP violation in CKM-favored  $B_s$  decays, the decay width of the CP-even lighter state  $L$  is greater than that of the CP-odd heavier state  $H$ . This property,  $\Delta\Gamma > 0$ , has been shown some time ago by explicit calculations of decays to CP-even and CP-odd eigenstates [17]. A recent calculation obtains  $\Delta\Gamma = 0.096 \pm 0.039 \text{ ps}^{-1}$  [18]. The average  $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$  is  $1/\bar{\tau}(B_s) = 1/(1.478_{-0.022}^{+0.020} \text{ ps})$  [16].

Defining a parameter  $\eta \equiv +1$  for a tagged  $B_s$  and  $-1$  for a tagged  $\bar{B}_s$ , we may write

$$\frac{d^4\Gamma[B_s(\bar{B}_s) \rightarrow (\ell^+\ell^-)_{J/\psi}(K^+K^-)_{\phi}]}{d\cos\theta_T d\phi_T d\cos\psi dt} \propto \frac{9}{32\pi} \{ [|A_0|^2 f_1(\vec{\rho}_T) + |A_{\parallel}|^2 f_2(\vec{\rho}_T)] \mathcal{T}_+ + |A_{\perp}|^2 f_3(\vec{\rho}_T) \mathcal{T}_- \\ + |A_{\parallel}||A_{\perp}| f_4(\vec{\rho}_T) \mathcal{U} + |A_0||A_{\parallel}| \cos(\delta_{\parallel}) f_5(\vec{\rho}_T) \mathcal{T}_+ + |A_0||A_{\perp}| f_6(\vec{\rho}_T) \mathcal{V} \} . \quad (6)$$

Here  $A_i \equiv A_i(t=0)$ , while dependence on time is given by the four functions

$$\mathcal{T}_{\pm} \equiv e^{-\Gamma t} [\cosh(\Delta\Gamma t/2) \mp \cos(2\beta_s) \sinh(\Delta\Gamma t/2) \mp \eta \sin(2\beta_s) \sin(\Delta m_s t)] , \quad (7)$$

$$\mathcal{U} \equiv e^{-\Gamma t} [\eta \sin(\delta_{\perp} - \delta_{\parallel}) \cos(\Delta m_s t) - \eta \cos(\delta_{\perp} - \delta_{\parallel}) \cos(2\beta_s) \sin(\Delta m_s t) \\ + \cos(\delta_{\perp} - \delta_{\parallel}) \sin(2\beta_s) \sinh(\Delta\Gamma t/2)] , \quad (8)$$

$$\mathcal{V} \equiv e^{-\Gamma t} [\eta \sin(\delta_{\perp}) \cos(\Delta m_s t) - \eta \cos(\delta_{\perp}) \cos(2\beta_s) \sin(\Delta m_s t) \\ + \cos(\delta_{\perp}) \sin(2\beta_s) \sinh(\Delta\Gamma t/2)] , \quad (9)$$

where relative strong phases are defined by

$$\delta_{\parallel} \equiv \text{Arg}(A_{\parallel}(0)A_0^*(0)) , \quad \delta_{\perp} \equiv \text{Arg}(A_{\perp}(0)A_0^*(0)) . \quad (10)$$

We now can see the source of the discrete two-fold ambiguity. The  $\cos\delta_{\parallel}$  term in the decay distribution and the time-dependent functions  $\mathcal{T}_{\pm}$ ,  $\mathcal{U}$ ,  $\mathcal{V}$  are invariant under the simultaneous substitutions [3]

$$2\beta_s \rightarrow \pi - 2\beta_s , \quad \Delta\Gamma \rightarrow -\Delta\Gamma , \quad \delta_{\parallel} \rightarrow -\delta_{\parallel} , \quad \delta_{\perp} \rightarrow \pi - \delta_{\perp} . \quad (11)$$

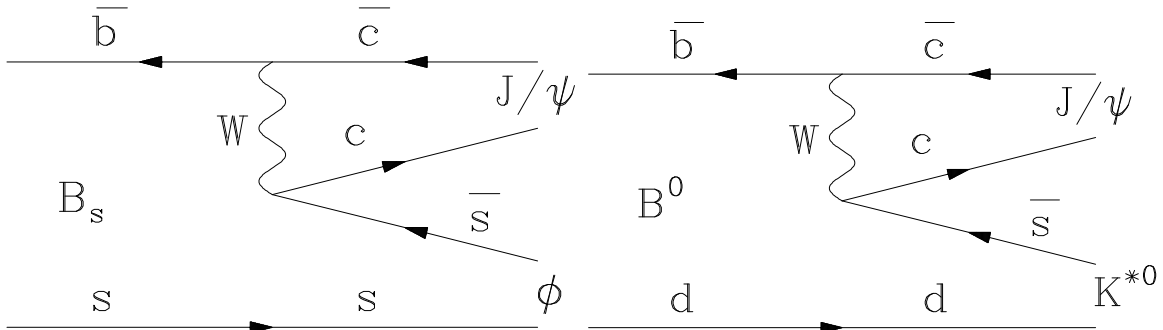


Figure 1: Color-suppressed tree diagrams, providing the dominant contribution to  $B_s \rightarrow J/\psi\phi$  (left) and  $B^0 \rightarrow J/\psi K^{*0}$  (right)

Without tagging information [i.e., setting  $\eta = 0$  in Eqs. (7–9)], there would be additional invariances under  $\beta_s \rightarrow -\beta_s$ ,  $\delta_{\parallel} \rightarrow -\delta_{\parallel}$ ,  $\delta_{\perp} \rightarrow \pi - \delta_{\perp}$  and under  $2\beta_s \rightarrow 2\beta_s - \pi$ ,  $\Delta\Gamma \rightarrow -\Delta\Gamma$ .

Magnitudes and phase amplitudes for  $B^0 \rightarrow J/\psi K^{*0}$  obtained by BaBar [19], Belle [20], the Heavy Flavor Averaging Group (HFAG) averages of these values [21], and CDF [22] are compared in Table I. These entail  $\sin(\delta_{\perp} - \delta_{\parallel}) \simeq -0.4$ ,  $\cos(\delta_{\perp} - \delta_{\parallel}) \simeq 0.9$ ,  $\sin(\delta_{\perp}) \simeq 0.2$ ,  $\cos(\delta_{\perp}) \simeq -0.98$ . A change in sign of  $\cos(\delta_{\perp} - \delta_{\parallel})$  and  $\cos(\delta_{\perp})$  has nearly maximal effect on the  $\sin(\Delta mt)$  and  $\sinh(\Delta\Gamma t/2)$  terms in  $\mathcal{U}$  and  $\mathcal{V}$  if not balanced by changes in sign of  $\cos(2\beta_s)$  and  $\Delta\Gamma$ . Hence such strong phases, if employed in fits to  $B_s \rightarrow J/\psi\phi$  tagged time-dependent decays, will be effective in resolving the twofold ambiguity in  $\beta_s$  and  $\Delta\Gamma$ . It is notable that magnitudes of amplitudes close to those quoted in Table I, and solutions for  $\delta_{\parallel}$  and  $\delta_{\perp}$  in ranges of values consistent with those in Table I, were obtained by the CDF Collaboration in a fit to *untagged*  $B_s \rightarrow J/\psi\phi$  decays [6, 22] and by the D0 Collaboration in a *flavor-tagged* study of this process [4].

We now discuss our reason for expecting strong phases in  $B_s \rightarrow J/\psi\phi$  very similar to those in  $B^0 \rightarrow J/\psi K^{*0}$ . Our conclusion is based on the high degree of similarity governing these two processes. They are dominated by the color-suppressed tree diagrams illustrated in Fig. 1. These two processes differ only by the substitution  $s \leftrightarrow d$  of the spectator quark. Indeed, they are characterized by similar branching ratios [2]:

$$\mathcal{B}(B_s \rightarrow J/\psi\phi) = (0.93 \pm 0.33) \times 10^{-3}, \quad \mathcal{B}(B^0 \rightarrow J/\psi K^{*0}) = (1.33 \pm 0.06) \times 10^{-3}. \quad (12)$$

(The former is based on a very old value [23] and deserves to be updated.) Taking account of the ratio of lifetimes [16]  $\tau(B_s)/\tau(B^0) = 0.966 \pm 0.015$ , one then finds the ratio of decay rates to be

$$\frac{\Gamma(B^0 \rightarrow J/\psi K^{*0})}{\Gamma(B_s \rightarrow J/\psi\phi)} = 1.38 \pm 0.49. \quad (13)$$

Additional processes contributing to both decays and differing only by the substitution  $s \leftrightarrow d$  of the spectator quark involve gluonic and electroweak penguin amplitudes, illustrated respectively in Figs. 2 and 3. One expects that the degree of flavor symmetry violation associated with the coherent sum of Figs. 1–3 will not be greater than the

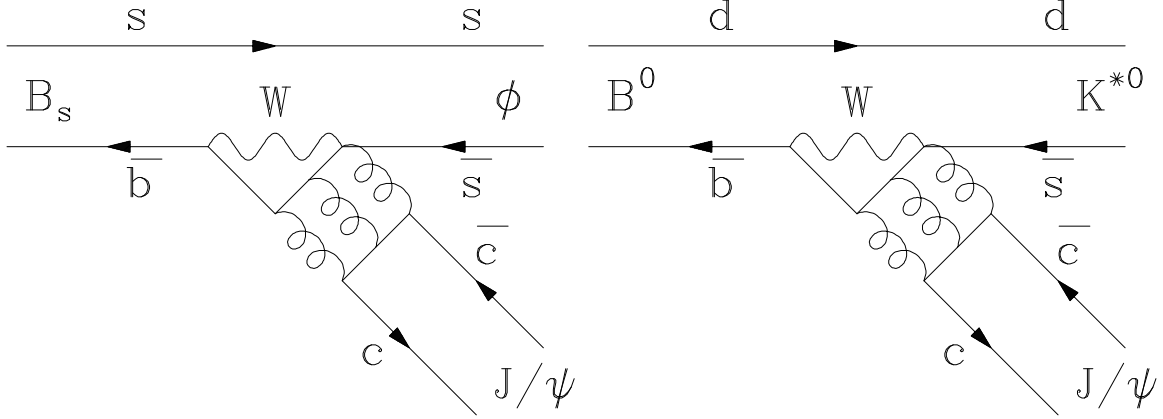


Figure 2: Gluonic penguin diagrams contributing to  $B_s \rightarrow J/\psi\phi$  (left) and  $B^0 \rightarrow J/\psi K^{*0}$  (right)

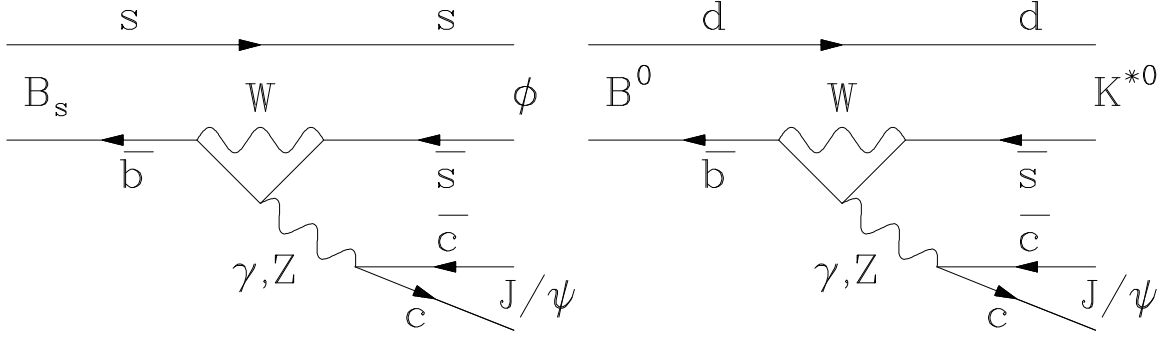


Figure 3: Electroweak penguin diagrams contributing to  $B_s \rightarrow J/\psi\phi$  (left) and  $B^0 \rightarrow J/\psi K^{*0}$  (right).

flavor violation in the individual components. Typical violations of this symmetry do not exceed 30% in the magnitudes of amplitudes.

Experimental evidence for approximate SU(3) invariance of both the magnitudes of amplitudes and their relative strong phases is provided by decay rates and CP asymmetries measured for  $B$  meson decays into  $\pi\pi, K\pi$  and  $K\bar{K}$  [24]. One particular test, which is sensitive to SU(3) invariance of relative strong phases, relates CP asymmetries in  $B^0 \rightarrow K^+\pi^-$  and  $B^0 \rightarrow \pi^+\pi^-$  [25, 26],

$$\frac{A_{CP}(B^0 \rightarrow K^+\pi^-)}{A_{CP}(B^0 \rightarrow \pi^+\pi^-)} = -\frac{\mathcal{B}(B^0 \rightarrow \pi^+\pi^-)}{\mathcal{B}(B^0 \rightarrow K^+\pi^-)} . \quad (14)$$

Experimentally the two ratios read [16]

$$-0.255 \pm 0.057 = -0.266 \pm 0.014 . \quad (15)$$

This shows that strong phases and not only magnitudes of amplitudes are approximately equal for SU(3)-related processes.

Special care must be taken when comparing  $B_s \rightarrow J/\psi\phi$  and  $B^0 \rightarrow J/\psi K^{*0}$  using flavor SU(3). In fact, here one is not using SU(3) but U(3), i.e., nonet symmetry. One

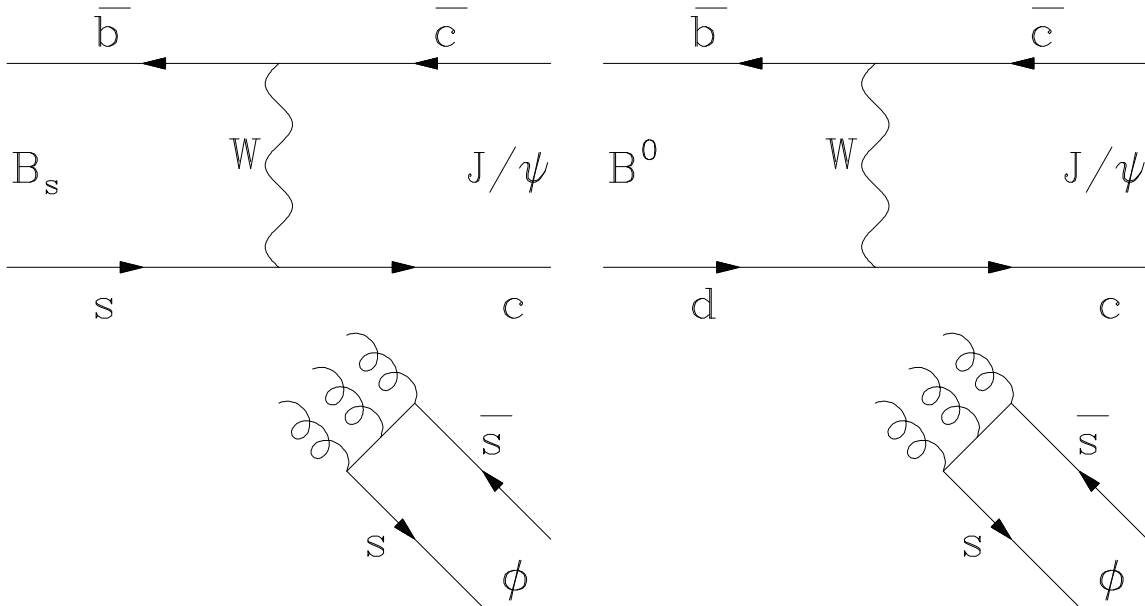


Figure 4: Left: exchange diagram contributing to  $B_s \rightarrow J/\psi\phi$ . No corresponding diagram can be drawn for  $B^0 \rightarrow J/\psi K^{*0}$ . Right: corresponding Cabibbo-suppressed diagram contributing to  $B^0 \rightarrow J/\psi\phi$ .

must investigate contributions unique to the SU(3)-singlet component of the  $\phi$ . The dominant diagram that contributes to  $B_s \rightarrow J/\psi\phi$  through the  $\phi$  singlet component, and not to  $B^0 \rightarrow J/\psi K^{*0}$ , is shown in Fig. 4 (left). Here the  $\phi$  couples to the rest of the  $W$ -exchange diagram via a minimum of three gluons, so this contribution is expected to be suppressed by the Okubo-Zweig-Iizuka (OZI) [27] rule. Another diagram, doubly-OZI-suppressed, involves annihilation of the  $b\bar{s}$  pair to a  $u, c, t$  loop (“penguin annihilation”), which is then connected to the  $c\bar{c}$  pair of the  $J/\psi$  and the  $s\bar{s}$  pair of the  $\phi$  each by a single gluon, with the  $c\bar{c}$  and  $s\bar{s}$  pair connected to each other by two more gluons. We expect this diagram to contribute even less than that in Fig. 4 (left) because of an extra loop factor in the penguin annihilation diagram.

A corresponding Cabibbo-suppressed  $W$ -exchange diagram, shown in Fig. 4 (right), (and a penguin annihilation diagram as mentioned above) can contribute to  $B^0 \rightarrow J/\psi\phi$ , for which a new upper bound is  $\mathcal{B}[B^0 \rightarrow J/\psi\phi] < 9.4 \times 10^{-7}$  [28]. We have recently found this diagram to contribute a rate smaller than the prediction  $\mathcal{B}(B^0 \rightarrow J/\psi\phi) = (1.8 \pm 0.3) \times 10^{-7}$  due to  $\omega$ - $\phi$  mixing [29]. We thus estimate that the exchange process illustrated in Fig. 4 (left) can contribute no more than  $[(1 - \frac{1}{2}\lambda^2)^2/\lambda^2][\tau(B_s)/\tau(B^0)]\mathcal{B}(B^0 \rightarrow J/\psi\phi) \simeq (3.2 \pm 0.5) \times 10^{-6}$  to  $\mathcal{B}(B_s \rightarrow J/\psi\phi)$ . This is only about 1/300 of its measured value.

We thus conclude that the similarity of amplitudes and strong phases for  $B_s \rightarrow J/\psi\phi$  and  $B^0 \rightarrow J/\psi K^{*0}$ , at least to the degree needed to eliminate the discrete ambiguity noted in Ref. [3], is a well-founded assumption. The fact that the  $\phi$  is a nonet, rather than octet, partner of  $K^{*0}$  is not an essential complication.

Because of the normalization (2), given magnitudes of transversity amplitudes are equivalent to given values for their ratios. Symmetry breaking factors which are common

to the three transversity amplitudes cancel in ratios of these amplitudes. Thus we expect symmetry relations between these ratios in  $B_s \rightarrow J/\psi\phi$  and  $B^0 \rightarrow J/\psi K^{*0}$  to hold more precisely than the equality  $\sqrt{\Gamma(B_s \rightarrow J/\psi\phi)} \simeq \sqrt{\Gamma(B^0 \rightarrow J/\psi K^{*0})}$ , which may involve a U(3)-breaking correction up to the level of 30%. Indeed, magnitudes of corresponding normalized transversity amplitudes measured for instance by the CDF Collaboration are equal within a few percent [6, 22],

$$|A_0| = \begin{cases} 0.729 \pm 0.015, & B_s \rightarrow J/\psi\phi \\ 0.754 \pm 0.008, & B^0 \rightarrow J/\psi K^* \end{cases}, \quad |A_{\parallel}| = \begin{cases} 0.480 \pm 0.029, & B_s \rightarrow J/\psi\phi \\ 0.459 \pm 0.015, & B^0 \rightarrow J/\psi K^* \end{cases}, \quad (16)$$

where the  $B_s \rightarrow J/\psi\phi$  values are quoted assuming  $\beta_s = 0$ . We expect the relative strong phases  $\delta_{\parallel}$  and  $\delta_{\perp}$  in  $B_s \rightarrow J/\psi\phi$  and  $B^0 \rightarrow J/\psi K^*$  to be equal within a similar precision, i.e., a few percent of  $\pi$  or  $\simeq 10^\circ$ . Applying such an error when assuming equal strong phases for these two processes in fits to data would be a reasonable assumption.

We close with some remarks regarding angular and time distributions.

(1) All the analyses of  $B_s \rightarrow J/\psi\phi$  and  $B^0 \rightarrow J/\psi K^{*0}$  display appreciable dependence of acceptance on the transversity azimuthal angle  $\phi_T$  [5–7, 15, 19, 22]. This is because the angle between the leptons and the helicity axis, and hence the fraction of events that will be selected by choosing leptons with selection cuts on kinematic variables, depends on  $\phi_T$ . In contrast, if one transforms the transversity angles  $(\theta_T, \phi_T)$  to an equivalent set of *helicity angles*  $(\theta_H, \phi_H)$ , one should expect the acceptance to depend less on  $\phi_H$ , and all its variation due to lepton selection criteria should be concentrated in  $\theta_H$ . The angle  $\phi_H$  (to be defined below) describes the relative orientations of the decay planes defined by the decay products of  $V_1 = J/\psi$  and  $V_2 = \phi$  and the helicity axis. As rotations by  $\phi_H$  only affect the relative orientations of these planes, they should not lead to variations in efficiency of lepton selection.

To define the helicity angles we identify

$$(\sin \theta_T \cos \phi_T, \sin \theta_T \sin \phi_T, \cos \theta_T) = (\cos \theta_H, \sin \theta_H \cos \phi_H, \sin \theta_H \sin \phi_H). \quad (17)$$

The angular functions  $g_j(\vec{\rho}_H) = f_j(\vec{\rho}_T)$  for  $\vec{\rho}_H \equiv (\theta_H, \phi_H, \psi)$  then become

$$\begin{aligned} g_1(\vec{\rho}_H) &= 2 \cos^2 \psi \sin^2 \theta_H, \quad g_2(\vec{\rho}_H) = \sin^2 \psi (1 - \sin^2 \theta_H \cos^2 \phi_H), \\ g_3(\vec{\rho}_H) &= \sin^2 \psi (1 - \sin^2 \theta_H \sin^2 \phi_H), \quad g_4(\vec{\rho}_H) = -\sin^2 \psi \sin^2 \theta_H \sin 2\phi_H, \\ g_5(\vec{\rho}_H) &= \frac{1}{\sqrt{2}} \sin 2\psi \sin 2\theta_H \cos \phi_H, \quad g_6(\vec{\rho}_H) = \frac{1}{\sqrt{2}} \sin 2\psi \sin 2\theta_H \sin \phi_H. \end{aligned} \quad (18)$$

In the helicity basis, the amplitudes  $A_{\parallel}$  and  $A_{\perp}$  are treated on an equal footing, and the similarity of acceptances for these two amplitudes then becomes manifest by the same dependence of  $g_2$  and  $g_3$  on  $\psi$  and  $\theta_H$ . The three terms  $|A_{\parallel}|^2 g_2 + |A_{\perp}|^2 g_3 + \text{Im}(A_{\parallel}^* A_{\perp}) g_4$  in the angular distribution (3) are invariant under  $\cos \phi_H \leftrightarrow \sin \phi_H$ ,  $A_{\parallel}^* \leftrightarrow A_{\perp}$ . As the angle  $\phi_H$  describes only the relative orientations of the decay planes defined by the decay products of  $V_1 = J/\psi$  and  $V_2 = \phi$ , one expects acceptance to depend weakly upon  $\phi_H$ , and to be similar for the amplitudes  $A_{\parallel}$  and  $A_{\perp}$ . The acceptance for  $A_0$  will be *different* from that for  $A_{\parallel}$  and  $A_{\perp}$ . We advocate attempting to extract the maximum

information from the latter two amplitudes, even though they account for less than half the decay intensity. (See Table I and corresponding values of  $|A_0|^2$  and  $|A_{\parallel}|^2$  in  $B_s \rightarrow J/\psi\phi$  obtained in Refs. [4,6].)

Instead of Eq. (6) the angular dependence is now given by the helicity functions  $g_j(\vec{\rho}_H)$ , while the time dependence involves the same transversity amplitudes  $A_i$  and time-dependent functions,  $\mathcal{T}_{\pm}, \mathcal{U}$  and  $\mathcal{V}$ ,

$$\frac{d^4\Gamma[B_s(\overline{B}_s) \rightarrow (\ell^+\ell^-)_{J/\psi}(K^+K^-)_{\phi}]}{d\cos\theta_H d\phi_H d\cos\psi dt} \propto \frac{9}{32\pi} \{ [|A_0|^2 g_1(\vec{\rho}_H) + |A_{\parallel}|^2 g_2(\vec{\rho}_H)] \mathcal{T}_+ + |A_{\perp}|^2 g_3(\vec{\rho}_H) \mathcal{T}_- \\ + |A_{\parallel}||A_{\perp}| g_4(\vec{\rho}_H) \mathcal{U} + |A_0||A_{\parallel}| \cos(\delta_{\parallel}) g_5(\vec{\rho}_H) \mathcal{T}_+ + |A_0||A_{\perp}| g_6(\vec{\rho}_H) \mathcal{V} \} . \quad (19)$$

To guard against systematic errors associated with incompletely understood acceptances, we suggest weighting the angular distribution by a function which emphasizes its dependence on  $A_{\parallel}$  and  $A_{\perp}$  and de-emphasizes its dependence on  $A_0$ . Such a function is

$$g(\cos\psi) = 3 - 5\cos^2\psi , \quad \int_{-1}^1 d(\cos\psi) g(\cos\psi) \begin{Bmatrix} \cos^2\psi \\ \sin^2\psi \\ \sin 2\psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{8}{3} \\ 0 \end{Bmatrix} . \quad (20)$$

Consequently,  $\int_{-1}^1 d(\cos\psi) g(\cos\psi) g_j(\vec{\rho}_H)$  is non-zero only for  $j = 2, 3, 4$ , corresponding to the three contributions involving only  $A_{\parallel}$  and  $A_{\perp}$ .

Integrating also over  $\theta_H$  one obtains

$$\int_{-1}^1 d(\cos\psi) g(\cos\psi) \frac{d^3\Gamma[B_s(\overline{B}_s) \rightarrow (\ell^+\ell^-)_{J/\psi}(K^+K^-)_{\phi}]}{d\phi_H d\cos\psi dt} \propto \\ \frac{1}{\pi} \left[ |A_{\parallel}|^2 \left( \frac{3}{2} - \cos^2\phi_H \right) \mathcal{T}_+ + |A_{\perp}|^2 \left( \frac{3}{2} - \sin^2\phi_H \right) \mathcal{T}_- - |A_{\parallel}||A_{\perp}| \sin 2\phi_H \mathcal{U} \right] . \quad (21)$$

The three trigonometric functions of  $\phi_H$  are linearly independent. This enables separate measurements of  $|A_{\parallel}|$ ,  $|A_{\perp}|$  and of the time-dependent functions,  $\mathcal{T}_+$ ,  $\mathcal{T}_-$  and  $\mathcal{U}$  for tagged  $B_s$  ( $\eta = +1$ ) and  $\overline{B}_s$  ( $\eta = -1$ ). This may be used to determine  $\beta_s$  and  $\Delta\Gamma$ .

(2) Integration of angular distributions with all angles except the transversity polar angle  $\theta_T$  was shown in Ref. [11] to separate CP-even contributions, behaving as  $1 + \cos^2\theta_T$ , from CP-odd contributions, behaving as  $\sin^2\theta_T$ . Dependence upon strong phases, contained only in interference terms which integrate to zero, then disappears. Important information is still retained in the functions  $\mathcal{T}_{\pm}$ , whose oscillatory behavior provides information on  $\beta_s$ . If  $\sin 2\beta_s$  is appreciable, as in the most recent fits to CDF and D0 data based on tagged  $B_s$  samples [3,4], this time-dependence should be clearly visible. As one example, we have chosen “best-fit” values  $\Delta\Gamma/\Gamma = 0.228$ ,  $2\beta_s = 0.77$  from Ref. [3], assumed a pessimistic dilution factor  $D = 0.11$  [3] to multiply the factor  $\eta$  in Eq. (7), and display the functions  $\mathcal{T}_{\pm}$  for  $B_s$  and  $\overline{B}_s$  tags. The results are shown in Fig. 5. If the oscillations are visible in fits to data, there should be no question about the observation of large  $\sin 2\beta_s$ . This method avoids possible biases involving the extraction of  $\beta_s$  from the interference terms  $\mathcal{U}$  and  $\mathcal{V}$  involving strong phases.

*Note added:* A month after the submission of this Letter the DO Collaboration reported new measurements of transversity amplitudes in  $B_s \rightarrow J/\psi\phi$  and  $B^0 \rightarrow J/\psi K^*$



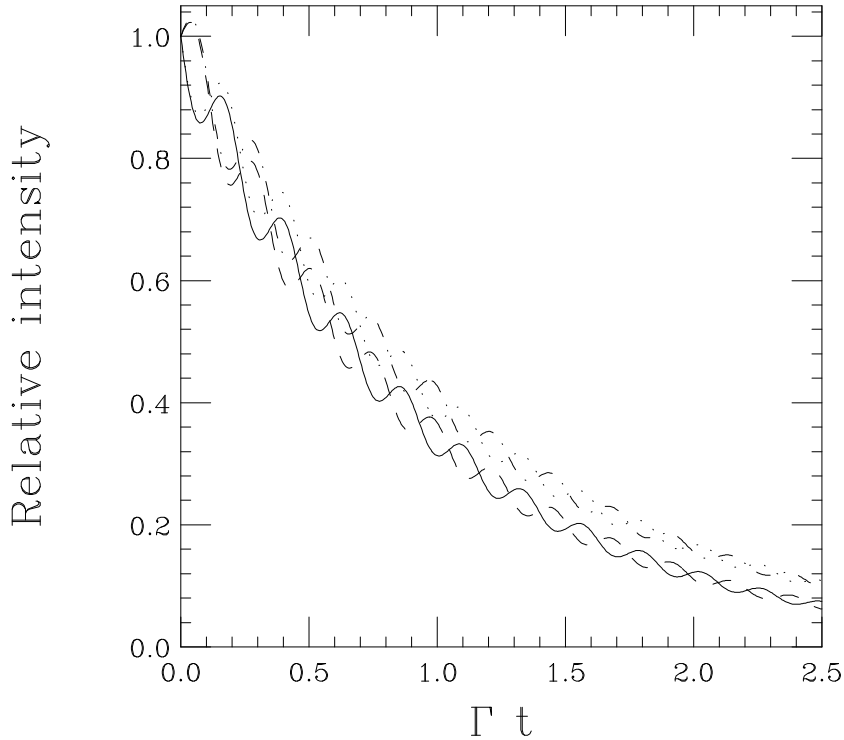


Figure 5: Time-dependence of  $\mathcal{T}_{\pm}$  for  $B_s$  and  $\overline{B}_s$  tags based on “best-fit” parameters for  $B_s \rightarrow J/\psi\phi$  decays from Ref. [3] (see text). Solid:  $\mathcal{T}_+$ ,  $B_s$  tag. Dashed:  $\mathcal{T}_+$ ,  $\overline{B}_s$  tag. Dot-dashed:  $\mathcal{T}_-$ ,  $B_s$  tag. Dotted:  $\mathcal{T}_-$ ,  $\overline{B}_s$  tag.

(assuming  $\beta_s = 0$  in the former) with values similar to the CDF measurements quoted in Eq. (16) [30]:

$$|A_0| = \begin{cases} 0.745 \pm 0.019, & B_s \rightarrow J/\psi\phi \\ 0.766 \pm 0.011, & B^0 \rightarrow J/\psi K^* \end{cases}, \quad |A_{\parallel}| = \begin{cases} 0.494 \pm 0.035, & B_s \rightarrow J/\psi\phi \\ 0.480 \pm 0.029, & B^0 \rightarrow J/\psi K^* \end{cases}. \quad (22)$$

This supports our assumption that the strong phase differences  $\delta_{\parallel}$  and  $\delta_{\perp}$  in these two processes are equal within  $10^\circ$ .

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## References

- [1] L. Wolfenstein, Phys. Rev. Lett. **51** (1983) 1945.
- [2] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667** (2008) 1.

- [3] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100** (2008) 161802 [arXiv:0712.2397 [hep-ex]].
- [4] V. M. Abazov *et al.* [D0 Collaboration], arXiv:0802.2255 [hep-ex].
- [5] D. E. Acosta *et al.* [CDF Collaboration], Phys. Rev. Lett. **94** (2005) 101803 [arXiv:hep-ex/0412057].
- [6] T. Aaltonen *et al.* [CDF collaboration], Phys. Rev. Lett. **100** (2008) 121803 [arXiv:0712.2348 [hep-ex]].
- [7] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **98** (2007) 121801 [arXiv:hep-ex/0701012].
- [8] I. Dunietz, R. Fleischer and U. Nierste, Phys. Rev. D **63** (2001) 114015 [arXiv:hep-ph/0012219].
- [9] A. S. Dighe, I. Dunietz and R. Fleischer, Phys. Lett. B **433** (1998) 147 [arXiv:hep-ph/9804254].
- [10] I. Dunietz, H. R. Quinn, A. Snyder, W. Toki and H. J. Lipkin, Phys. Rev. D **43** (1991) 2193.
- [11] A. S. Dighe, I. Dunietz, H. J. Lipkin and J. L. Rosner, Phys. Lett. B **369** (1996) 144 [arXiv:hep-ph/9511363].
- [12] J. L. Rosner, Phys. Rev. D **42** (1990) 3732.
- [13] R. Fleischer and I. Dunietz, Phys. Rev. D **55** (1997) 259 [arXiv:hep-ph/9605220].
- [14] A. S. Dighe, I. Dunietz and R. Fleischer, Eur. Phys. J. C **6** (1999) 647 [arXiv:hep-ph/9804253].
- [15] CDF Collaboration, Public Note CDF/ANAL/BOTTOM/PUBLIC/9458 (unpublished).
- [16] E. Barberio *et al.*, [Heavy Flavor Averaging Group (HFAG) Collaboration] arXiv:0808.1297 [hep-ex].
- [17] R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Lett. B **316** (1993) 567.
- [18] A. Lenz and U. Nierste, JHEP **0706** (2007) 072 [arXiv:hep-ph/0612167].
- [19] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **76** (2007) 031102 [arXiv:0704.0522 [hep-ex]].
- [20] R. Itoh *et al.* [Belle Collaboration], Phys. Rev. Lett. **95** (2005) 091601 [arXiv:hep-ex/0504030].
- [21] Ref. [16], p. 159.

- [22] CDF Collaboration, Public Note CDF/ANAL/BOTTOM/PUBLIC/8950 (unpublished).
- [23] F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **54** (1996) 6596 [arXiv:hep-ex/9607003].
- [24] C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, Phys. Rev. D **70** (2004) 034020 [arXiv:hep-ph/0404073].
- [25] N. G. Deshpande and X. G. He, Phys. Rev. Lett. **75** (1995) 1703 [arXiv:hep-ph/9412393].
- [26] M. Gronau and J. L. Rosner, Phys. Rev. Lett. **76** (1996) 1200 [arXiv:hep-ph/9510363].
- [27] S. Okubo, Phys. Lett. **5** (1963) 165; G. Zweig, CERN Report No. 8419/TH-412 (1964); J. Iizuka, Prog. Theor. Phys. Suppl. **37** (1966) 21.
- [28] Y. Liu *et al.* [Belle Collaboration], Phys. Rev. D **78** (2008) 011106 [arXiv:0805.3225 [hep-ex]].
- [29] M. Gronau and J. L. Rosner, Phys. Lett. B **666** (2008) 185 [arXiv:0806.3584 [hep-ph]].
- [30] V. M. Abazov *et al.* [D0 Collaboration], arXiv:0810.0037 [hep-ex].